Cl-XI. (Maths)

Trigonometrical Ratios of the Acute angles and Associated angles.

S.N.-1 If $\frac{\cos^4 x}{\cos^2 y} + \frac{\sin^4 x}{\sin^2 y} = 1$, prove that $\frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x} = 1$.

Sol^n. $\frac{\cos^4 x}{\cos^2 y} + \frac{\sin^4 x}{\sin^2 y} = 1 \Rightarrow \frac{\cos^4 x}{\cos^2 y} + \frac{\sin^4 x}{\sin^2 y} = \cos^2 x + \sin^2 x$.

$$\Rightarrow \cos^2 x\left[\frac{\cos^2 x}{\cos^2 y} - 1\right] = \sin^2 x\left[1 - \frac{\sin^2 x}{\sin^2 y}\right]$$

$$\Rightarrow \frac{\cos^2 x}{\cos^2 y}\left[\cos^2 x - \cos^2 y\right] = \frac{\sin^2 x}{\sin^2 y}\left[\sin^2 y - \sin^2 x\right]$$

$$\Rightarrow \frac{\cos^2 x}{\cos^2 y}\left[\cos^2 x - \cos^2 y\right] = \frac{\sin^2 x}{\sin^2 y}\left[\cos^2 x - \cos^2 y\right]$$

$$\Rightarrow \frac{\cos^2 x}{\cos^2 y} = \frac{\sin^2 x}{\sin^2 y} = \frac{\cos^2 x + \sin^2 x}{\cos^2 y + \sin^2 y} = 1.$$

Now

$$L.H.S = \frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x} = \cos^2 y + \sin^2 y = 1 = R.H.S.$$

[putting $\cos^2 x = \cos^2 y$, $\sin^2 x = \sin^2 y$]

S.N-2 If $\frac{\sin^4\theta}{x} + \frac{\cos^4\theta}{y} = \frac{1}{x+y}$ then show that $\frac{\sin^{12}\theta}{x^5} + \frac{\cos^{12}\theta}{y^5} = \frac{1}{(x+y)^5}$

Sol^n $\frac{\sin^4\theta}{x} + \frac{\cos^4\theta}{y} = \frac{1}{x+y}$

$$\Rightarrow \frac{\sin^4\theta}{x} - \frac{\sin^2\theta}{x+y} = \frac{\cos^2\theta}{x+y} - \frac{\cos^4\theta}{y} \Rightarrow \frac{(x+y)\sin^4\theta - x\sin^2\theta}{x(x+y)} = \frac{y\cos^2\theta - (x+y)\cos^4\theta}{y(x+y)}$$

$$\Rightarrow \frac{y\sin^4\theta - x\sin^2\theta(1-\sin^2\theta)}{x} = \frac{y\cos^2\theta(1-\cos^2\theta) - x\cos^4\theta}{y}$$

$$\Rightarrow \frac{\sin^2\theta\left(y\sin^2\theta - x\cos^2\theta\right)}{x} = \frac{\cos^2\theta\left(y\sin^2\theta - x\cos^2\theta\right)}{y}$$

$$\Rightarrow \frac{\sin^2\theta}{x} = \frac{\cos^2\theta}{y} = \frac{1}{x+y}.$$

$$\therefore \sin^2\theta = \frac{x}{x+y}, \quad \cos^2\theta = \frac{y}{x+y}.$$

$$\therefore L.H.S = \frac{\sin^{12}\theta}{x^5} + \frac{\cos^{12}\theta}{y^5} = \frac{1}{x^5}\left(\frac{x}{x+y}\right)^6 + \frac{1}{y^5}\left(\frac{y}{x+y}\right)^6$$

$$= \frac{x}{(x+y)^6} + \frac{y}{(x+y)^6} = \frac{x+y}{(x+y)^6} = \frac{1}{(x+y)^5} = R.H.S$$

Cl - XI (Maths).

SN-3 prove that $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]=1$

Sol^n.

$\therefore$ L.H.S $=3\left[(-\cos\alpha)^4+(-\sin\alpha)^4\right]-2\left[(\cos\alpha)^6+(\sin\alpha)^6\right]$

$=3\left[(\cos^2\alpha+\sin^2\alpha)^2-2\cos^2\alpha\sin^2\alpha\right]-2\left[(\cos^2\alpha+\sin^2\alpha)^3-3\sin^2\alpha\cos^2\alpha(\cos^2\alpha+\sin^2\alpha)\right]$

$=3-\cancel{6\cos^2\alpha\sin^2\alpha}-2+\cancel{6\cos^2\alpha\sin^2\alpha}=1=$ RHS.

SN-4 Find the value of $\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$

Sol^n. Give that $\cos^2\left(\frac{\pi}{2}-\frac{3\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\cos^2\frac{5\pi}{8}+\cos^2\left(\frac{3\pi}{2}-\frac{5\pi}{8}\right)$

$=\sin^2\frac{3\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\sin^2\frac{5\pi}{8}=1+1=2$

SN-5 prove that $\tan^2\frac{\pi}{16}\tan^2\frac{2\pi}{16}\tan^2\frac{3\pi}{16}\tan^2\frac{4\pi}{16}\tan^2\frac{5\pi}{16}\tan^2\frac{6\pi}{16}\tan^2\frac{7\pi}{16}=1.$

Sol^n. L.H.S $=\left(\tan^2\frac{\pi}{16}\tan^2\frac{7\pi}{16}\right)\cdot\left(\tan^2\frac{2\pi}{16}\cdot\tan^2\frac{6\pi}{16}\right)\left(\tan^2\frac{3\pi}{16}\times\tan^2\frac{5\pi}{16}\right)\tan^2\frac{4\pi}{16}$

$=\left\{\tan^2\frac{\pi}{16}\times\tan^2\left\{\frac{\pi}{2}-\frac{\pi}{16}\right\}\right\}\times\left\{\tan^2\frac{2\pi}{16}\cdot\tan^2\left(\frac{\pi}{2}-\frac{2\pi}{16}\right)\right\}\times\left\{\tan^2\frac{3\pi}{16}\tan^2\left(\frac{\pi}{2}-\frac{3\pi}{16}\right)\tan^2\frac{4\pi}{16}\right\}.$

$=\tan^2\frac{\pi}{16}\cot^2\frac{\pi}{16}\times\tan^2\frac{2\pi}{16}\cot^2\frac{2\pi}{16}\times\tan^2\frac{3\pi}{16}\cot^2\frac{3\pi}{16}\times\tan^2\frac{4\pi}{16}$

$=\tan^2\frac{4\pi}{16}=1=$ RHS.

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S.N-6 Find the least value of $2^{\sin^2\theta} + 2^{\cos^2\theta}$.

Solⁿ. we know that AM ≥ GM.

$$\therefore \frac{2^{\sin^2\theta} + 2^{\cos^2\theta}}{2} \geq \sqrt{2^{\sin^2\theta} \cdot 2^{\cos^2\theta}} = \sqrt{2^{\sin^2\theta + \cos^2\theta}} = \sqrt{2}$$

$$\therefore 2^{\sin^2\theta} + 2^{\cos^2\theta} \geq 2\sqrt{2} \text{ proved.}$$

S.N-7. prove that $\tan\frac{\pi}{12} \tan\frac{5\pi}{12} \tan\frac{7\pi}{12} \tan\frac{11\pi}{12} = 1$.

$$L.H.S = \tan\frac{\pi}{12} \tan\frac{5\pi}{12} \tan\frac{7\pi}{12} \tan\frac{11\pi}{12}$$

$$= \tan\frac{\pi}{12} \tan\frac{5\pi}{12} \tan\left(\pi - \frac{5\pi}{12}\right) \tan\left(\pi - \frac{\pi}{12}\right)$$

$$= \tan\frac{\pi}{12} \tan\frac{5\pi}{12} \left(-\tan\frac{5\pi}{12}\right) \tan\left(-\frac{\pi}{12}\right)$$

$$= \tan^2\frac{\pi}{12} \tan^2\frac{5\pi}{12} = \tan^2\frac{\pi}{12} \tan^2\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$$

$$= \tan^2\frac{\pi}{12} \cot^2\frac{\pi}{12} = 1 = R.H.S.$$

S.N-8.

Find the sum of n terms:- $\sin\theta + \sin(\pi+\theta) + \sin(2\pi+\theta) + \cdots$

Solⁿ. when n is even,

$$\sin\theta + \sin(\pi+\theta) + \sin(2\pi+\theta) + \cdots + \sin\{(n-1)\pi + \theta\}$$

$$= \sin\theta + (-\sin\theta) + \sin\theta + \cdots + (-\sin\theta)$$

$$= 0.$$

when n is odd.

$$\sin\theta + \sin(\pi+\theta) + \sin(2\pi+\theta) + \cdots + \sin\{(n-1)\pi + \theta\}$$

$$= \sin\theta - \sin\theta + \sin\theta + (-\sin\theta) + \cdots + \sin\theta$$

$$= +\sin\theta.$$