SN-1

If in the triangle ABC, the side c and the angle C remain unchanged while the other sides and angles are changed slightly, show that

$$\frac{da}{\cos A} + \frac{db}{\cos B} = 0$$

Sol^n. we know that

$$\frac{a}{\sin A} = 2R$$

$$\therefore\ a = 2R \sin A$$

$$\therefore\ da = 2R\cos A\, dA$$

$$\therefore\ \frac{da}{\cos A} = 2R\,dA \quad \text{---(1)}$$

Similarly $\dfrac{db}{\cos B} = 2R\,dB$ ---(2)

$\therefore$ Adding (1) and (2), we get $\dfrac{da}{\cos A} + \dfrac{db}{\cos B} = 2R\,dA + 2R\,dB$

$$= 2R\,d(A+B)$$

$$= 2R\,d(\pi - c) = 0$$

SN-2

In the triangle ABC, the a, b remain constant but base angles A and B vary, then show that

$$\frac{dA}{\sqrt{a^2 - b^2\sin^2 A}} = \frac{dB}{\sqrt{b^2 - a^2\sin^2 B}}$$

Sol^n. we know that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = 2R$

$$\therefore\ a = \sin A \cdot 2R, \quad b = 2R\sin B$$

Again $a \sin B = b \sin A$

$$\therefore\ a\cos B\, dB = b\cos A\, dA$$

$$\therefore\ \frac{dA}{\sqrt{a^2\cos^2 B}} = \frac{dB}{\sqrt{b^2\cos^2 A}}$$

or $$\frac{dA}{\sqrt{a^2 - a^2\sin^2 B}} = \frac{dB}{\sqrt{b^2 - b^2\sin^2 A}}$$

or $$\frac{dA}{\sqrt{a^2 - b^2\sin^2 A}} = \frac{dB}{\sqrt{b^2 - a^2\sin^2 B}}$$

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SN-3. The period of oscillation T of a simple pendulum of l is connected by the relation $T = 2\pi\sqrt{l/g}$, where g is a constant. Find approximately the percentage error in the computed value of T corresponding to an error of 1% in the value of l.

Soln.

$T = 2\pi\sqrt{\frac{l}{g}} \quad \therefore \frac{dT}{dl} = 2\pi \cdot \frac{1}{2\sqrt{g}\sqrt{l}} \quad \therefore dT = \frac{\pi}{\sqrt{gl}}\Delta l$

$\therefore \frac{\Delta l}{l} \times 100 = 1 \quad \therefore \Delta l = .01 l$

$\therefore dT = \frac{\pi}{\sqrt{gl}} \times .01 l = 2\pi\sqrt{\frac{l}{g}} \times \frac{.01}{2} = T\frac{.01}{2}$

$= T \times .005$

$\therefore \left(\frac{dT}{T} \times 100\right)\% = .5\%$

SN-4 Q units of heat is required to raise the temperature of 1 gm water from 0°c to t°c where

$Q = t + 10^{-5} \times 2t^2 + 10^{-7} \times 3t^3$

If the specific heat is the rate of increase of heat per unit degree rise of temperature, find the specific heat of water at 50°C.

Soln.

$Q = t + 10^{-5} \times 2t^2 + 10^{-7} \times 3t^3$

$\therefore \frac{dQ}{dt} = 1 + 4t \times 10^{-5} + 9t^2 \times 10^{-7}$

$\therefore$ ~~At 50~~ specific heat of ~~temperature~~ water at 50°c

$\therefore \left[\frac{dQ}{dt}\right]_{t=50} = 1 + 10^{-5} \times 200 + 10^{-7} \times 9 \times 50^2$

$= 1 + .002 + .00225$

$= 1.00425$

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SN-5

The base of a water tank is a square of side 3 ft. Find the rate of flow of water in the tank, if the water level rises at the rate of 1 ft/min.

Soln.

Suppose that height and length of the water tank be h and r.

$\therefore V = hr^2 \quad \therefore \frac{dV}{dt} = r^2 \frac{dh}{dt} = 3^2 \times 1 = 9.$

$\therefore$ Rate of flow of water = 9 cu ft/min.

S.N-6

The circular ink blot grows at the rate of 2 cm²/sec. Find the rate at which the radius is increasing after $2\frac{6}{11}$ secs.

Soln.

Suppose radius = R after $2\frac{6}{11}$ sec.

$\therefore$ A.T.P. $\pi R^2 = 2 \times \frac{28}{11}$

or $R = \sqrt{\frac{2 \times 7 \times 28}{22 \times 11}} = \frac{14}{11}$

$\therefore S = \pi r^2$

$\frac{dS}{dt} = 2\pi r \frac{dr}{dt}$

or $2 = 2\pi r \frac{dr}{dt}$

or $\frac{dr}{dt} = \frac{1}{\pi r} = \frac{7 \times 11}{22 \times 14} = \frac{1}{4} = 0.25$

The radius is increasing .25 cm after $2\frac{6}{11}$ secs.

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