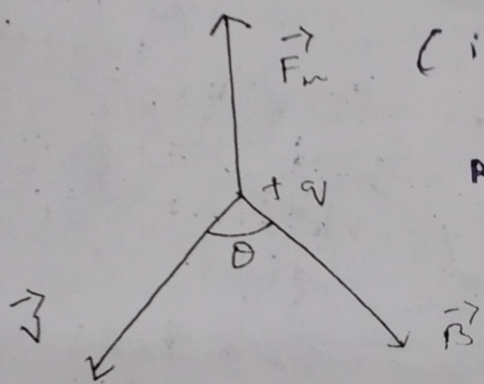
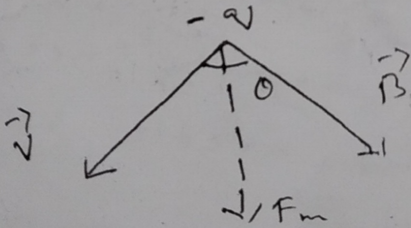
Lorentz Force: When a charge particle having charge q moves in a region where both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, it experiences a net force called Lorentz force.

$$\vec{F} = \vec{F}_e + \vec{F}_m$$
$$= q\vec{E} + q(\vec{v} \times \vec{B})$$
$$= q[\vec{E} + (\vec{v} \times \vec{B})]$$

where velocity $\vec{v}$ makes an angle θ with $\vec{B}$



(is $\perp$ to the plane containing $\vec{v}$ and $\vec{B}$
Right hand screw rule or right hand thumb rule)



$$|F_m| = Bqv\sin\theta$$

ⓐ Special cases: $\theta = 0°$ or $\theta = 180°$
i.e. q moves parallel or antiparallel to the direction of magnetic field $\vec{B}$, $F_m = 0$

ⓑ if $\theta = 90°$, i.e. charge moves $\perp$ to $\vec{B}$
$$F_m = Bqv\sin\theta = Bqv$$

If the charge is at rest,
$\vec{v} = 0$, $F_m = 0$

Def^n of Magnetic Field:

$$F_m = qvB\sin\theta$$

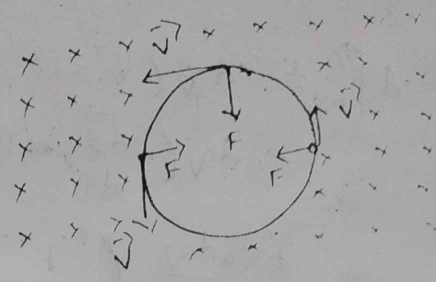
$$B = \frac{F_m}{qv\sin\theta}$$

$B = F_m$, when $q=1$, $v=1$ and $\theta = 90°$.

Thus magnetic field (B) at a point may be defined as, the magnetic force experienced by a unit charge moving with unit velocity at right angle to the magnetic field.

$$B = \frac{[MLT^{-2}]}{[AT\,LT^{-1}]} = [ML^0T^{-2}A^{-1}]$$

Charged particle Moving in a uniform Magnetic field



Magnetic field is uniform acting $\perp$ to the plane of paper.

Charge particle of $+q$ charge enter the region with velocity $\vec{v}$ and $\perp$ to $\vec{B}$

Magnetic Lorentz force $= F_m = q(\vec{v} \times \vec{B})$
$= qvB\sin\theta$
$= qvB\sin 90^\circ = qvB$

This force is $\perp$ to the direction of motion and $\vec{B}$. Thus the path ~~of~~ ~~path~~ is circular.

~~not~~ The centripetal force is provided by magnetic Lorentz force

$$\frac{mv^2}{r} = Bqv$$

or $r = \frac{mv}{Bq}$

$$T = \frac{2\pi r}{v} = \cancel{\frac{2\pi m}{}}$$

$$= \frac{2\pi}{v}\left(\frac{mv}{Bq}\right)$$

$$= \frac{2\pi m}{Bq}$$

Frequency $= f = \nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

Angular frequency

$$\omega = \frac{2\pi}{T}$$

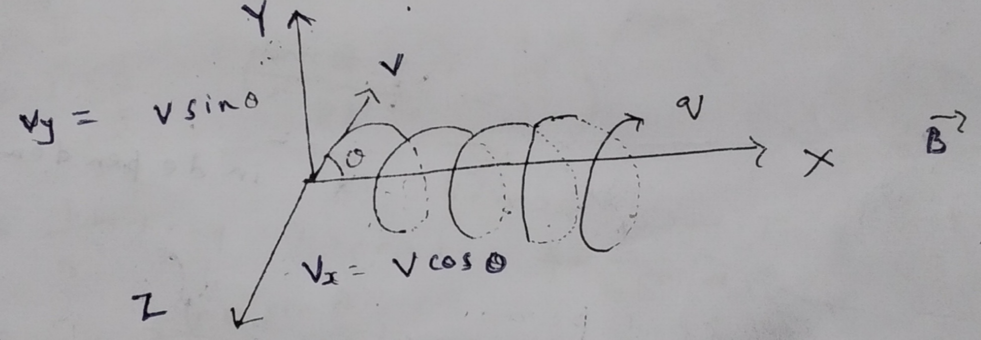
$$= 2\pi\left(\frac{Bq}{2\pi m}\right)$$

$$= \frac{Bq}{m}$$

Thus, angular frequency does n't depend upon the speed of the particle (provided speed of particle is very small as compared to the speed of light)

Thus all the charged particles take the same time to complete small or large circles, provided their specific charge (q/m) is same.

When a charged particle moves at an angle to the magnetic field (other than 0°, 90° and 180°)



Consider a charged particle of mass m and moving with velocity $\vec{v}$ at an angle θ with the direction of the magnetic field.

The charged particle moves with constant velocity $v\cos\theta$ along the magnetic field (as no force acts on the charged particle when it moves parallel to the magnetic field)

$V\sin\theta$ is $\perp$ to the dirt^n of $\vec{B}$, so the particle experiences a force. Thus charged particle tends to move in a circular path centripetal force is provided by the magnetic Lorentz force

$$\frac{m(v\sin\theta)^2}{r} = q(v\sin\theta)B\sin 90^\circ$$

$$\frac{m(v\sin\theta)^2}{r} = q(v\sin\theta)B$$

or $$r = \frac{mv\sin\theta}{Bq}$$

$$T = \frac{2\pi r}{v\sin\theta} = \left(\frac{2\pi}{v\sin\theta}\right) r$$

$$= \frac{2\pi}{v\sin\theta}\left(\frac{mv\sin\theta}{Bq}\right)$$

$$= \frac{2\pi m}{Bq}$$

Time period is independent of velocity of charged particle

$$\text{Frequency} = \nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

Pitch: The linear distance travelled by the charged particle in one rotation is called pitch of the helix

$$\text{Pitch} = (v\cos\theta)\times T$$

$$= v\cos\theta \times \left(\frac{2\pi m}{Bq}\right)$$

$$= \frac{2\pi m v\cos\theta}{Bq}$$

pitch

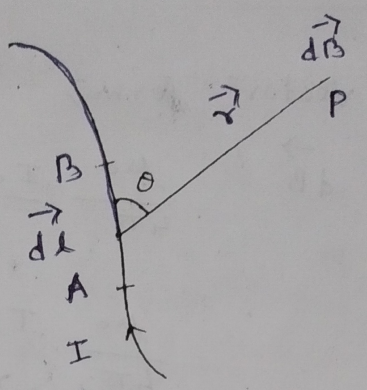
$$= V_x \times T$$

$$= V_x \times \left(\frac{2\pi r}{V_y}\right)$$

$$= \frac{V_x}{V_y}(2\pi r)$$

Biot Savart Law

Biot Savart law is used to determine the strength of magnetic field at any point due to current carrying conductor



Consider a small element AB of length dl carrying current I. Strength of magnetic field $\vec{dB}$ at a point P, distant r from the element is

$$dB \propto dl$$
$$dB \propto I$$
$$dB \propto \sin\theta \qquad (\theta \text{ is the angle bet}^{n}\ \vec{dl} \text{ and } \vec{r})$$

$$dB \propto \frac{1}{r^2}$$

i.e. $$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

$$dB = K\frac{I\,dl\sin\theta}{r^2}$$, where K is a constant of proportionality

$$K = \frac{\mu_0}{4\pi},$$

μ_0 is absolute permeability of free space

$$\mu_0 = 4\pi\times10^{-7}\ TmA^{-1} \text{ or } Wbm^{-1}A^{-1}$$

$$\therefore \frac{\mu_0}{4\pi} = 10^{-7}\ TmA^{-1}$$

In vector form,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3}$$

$$|\vec{dB}| = \frac{\mu_0 I |\vec{dl} \times \vec{r}|}{4\pi r^3}$$

The resultant magnetic field at a point P due to the whole conductor is

$$\vec{B} = \int \vec{dB} = \frac{\mu_0}{4\pi} \int \frac{I(\vec{dl} \times \vec{r})}{r^3}$$

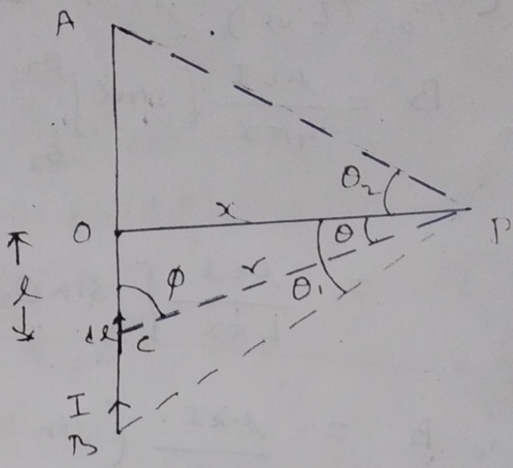
Direction of $\vec{dB}$ is same as $\vec{dl} \times \vec{r}$, it will be into the plane of paper $\otimes$

(* $\vec{r} \times \vec{dl} \rightarrow$ out of paper, R.H. Thumb rule)

[* permeability is the capability of a substance or a medium to have magnetisation in magnetic field. It indicates the degree or extent to which magnetic field lines can enter a substance. It is denoted by μ

Permeability $= \mu = \mu_0 \mu_r$, where
$\mu_0 =$ absolute permeability of free space
and μ_r is relative permeability of the medium

Magnetic Field due to Infinitely long straight wire carrying current using Biot Savart Law



consider a long straight wire AB carrying current I

Let r be distance of point P from the current element dl

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2} \quad -(1)$$

In Δ POC

$$\sin\phi = \frac{x}{r} = \cos\theta \quad -(2)$$

$$\text{or} \quad r = \frac{x}{\cos\theta} \quad -(3)$$

Now $\quad \tan\theta = \frac{l}{x}$

$$l = x\tan\theta$$

$$dl = x\sec^2\theta\, d\theta \quad -(4)$$

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{I\,(x\sec^2\theta\, d\theta)\cos\theta}{\left(\frac{x}{\cos\theta}\right)^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\cos\theta\, d\theta}{x}$$

Magnetic field due to whole conductor AB is

$$B = \int_{-\theta_1}^{\theta_2} dB = \frac{\mu_0 I}{4\pi x}\int_{-\theta_1}^{\theta_2}\cos\theta\, d\theta$$

(* By convention, θ_1 is anticlockwise and taken as -ve)

$$B = \frac{\mu_0 I}{4\pi x}\left[\sin\theta\right]_{-\theta_1}^{\theta_2}$$

$$= \frac{\mu_0 I}{4\pi x}\left[\sin\theta_2 - \sin(-\theta_1)\right]$$

$$\boxed{B = \frac{\mu_0 I}{4\pi x}\left(\sin\theta_1 + \sin\theta_2\right)}$$

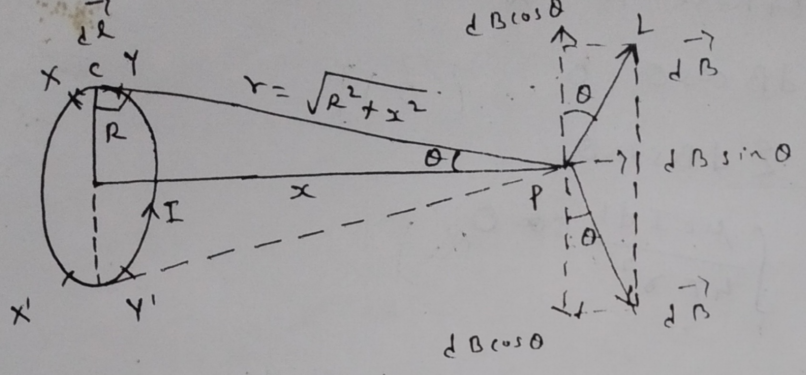
If the straight wire is infinitely long, then,

$$B = \frac{\mu_0 I}{4\pi x}\left(\sin 90^\circ + \sin 90^\circ\right)$$

$$\frac{\mu_0}{4\pi}\left(\frac{2I}{x}\right)$$

Direction of magnetic field at point P will be $\perp$ to the plane containing $\vec{dl}$ and $\vec{r}$ and is directed into the plane of paper $\otimes$

Magnetic Field on the Axis of a circular loop (or a Ring or a coil) carrying current -



Let XY be a small element of length dl at a distance r from point P.
(Every current element is $\perp$ to $\hat{r}$ (showing the direction of $\vec{r}$))

Magnetic field due to a small element XY at point P is

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\phi}{r^2}$$, where

ϕ is the angle betn dl and r $= 90°$

$$\therefore \quad dB = \frac{\mu_0 I\,dl}{4\pi r^2}$$

Direction of $\vec{dB}$ is $\perp$ to the plane formed by $\vec{dl}$ and $\vec{r}$ and is along PL which is $\perp$ to PC.

Resolving dB into two components

(i) $dB\cos\theta \rightarrow$ which is $\perp$ to axis of coil

(ii) $dB\sin\theta \rightarrow$ which is along the axis of coil away from the centre of coil.

coil is symmetrical about its axis, for every XY, there is $X'Y'$

$$\therefore \quad \Sigma \, dB \cos\theta = 0$$

$$B = \Sigma \, dB \sin\theta$$

$$= \int \frac{\mu_0 I \, dl}{4\pi r^2} \sin\theta$$

$$B = \frac{\mu_0 I \sin\theta}{4\pi r^2} \int dl$$

$$\int dl = 2\pi R$$

$$\therefore \quad B = \frac{\mu_0 I \sin\theta \times 2\pi R}{4\pi r^2}$$

Now $\sin\theta = \frac{R}{r} = \frac{R}{\sqrt{R^2 + x^2}}$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{2\pi I R^2}{r^3}$$

$$= \frac{\mu_0}{4\pi} \frac{2\pi I R^2}{(R^2 + x^2)^{3/2}}$$

If the coil has n turns

$$B' = nB = \frac{\mu_0}{4\pi} \frac{2\pi n I R^2}{(R^2 + x^2)^{3/2}}$$

Magnetic field at the centre of the coil, put $x = 0$

$$B' = \frac{\mu_0}{4\pi} \frac{2\pi n I}{R}$$