Class - IX

Coordinate Geometry : Internal and External Division of Straight line Segment

Let's Calculate - 19

Class - (IX)

Let us Calculate - 19

4) Let us calculate in what ratio is the line segment joining the points (7, 3) and (-9, 6) divided by the y axis.

(Ans) Let y axis divides the line joining (7, 3) and (-9, 6) into m : n ratio at P, then coordinate of P is

$$\equiv \left(\frac{m\times(-9)+n\times 7}{m+n}, \frac{m\times 6+n\times 3}{m+n}\right)$$

$$\equiv \left(\frac{7n-9m}{m+n}, \frac{6m+3n}{m+n}\right)$$

As P lies on y axis therefore it's x cordinate = 0

$$\therefore \frac{7n-9m}{m+n} = 0 \Rightarrow 7n = 9m \Rightarrow \frac{m}{n} = 7/9$$

$\therefore m : n = 7 : 9$

$\therefore$ y axis divides the line into 7 : 9 ratio

5) Prove that when the points A (7, 3), B (9, 6) C (10, 12) and D (8, 9) are joined in order, then they will form a parallelogram

(Ans) $AB = \sqrt{(7-9)^2+(3-6)^2}$

$= \sqrt{(-2)^2+(-3)^2} = \sqrt{13}$

$BC = \sqrt{(9-10)^2+(6-12)^2} = \sqrt{1+36} = \sqrt{37}$

$CD = \sqrt{(10-8)^2+(12-9)^2} = \sqrt{2^2+3^2} = \sqrt{13}$

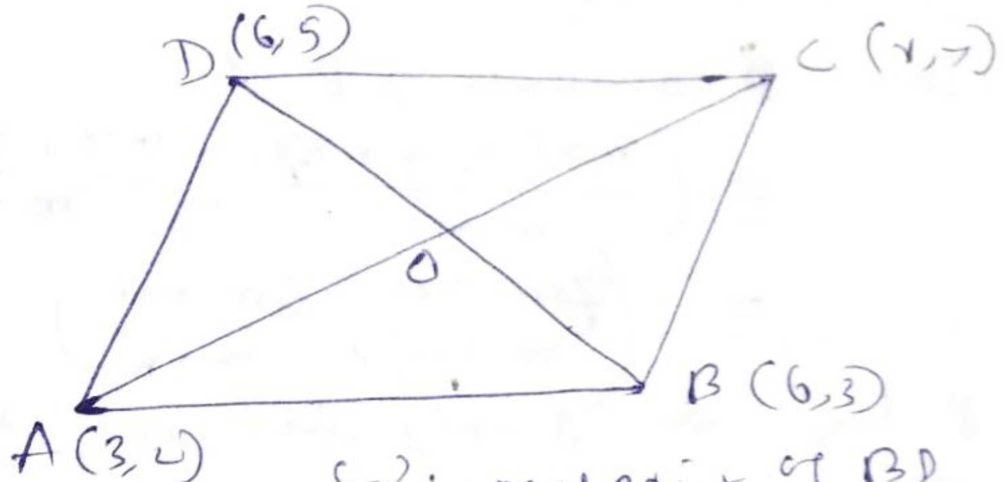
$DA = \sqrt{(8-7)^2+(9-3)^2} = \sqrt{1+36} = \sqrt{37}$

$AB = CD$ & $BC = DA$

i.e. as opposite sides are equal, $\therefore$ ABCD is Parallelogram

6) If the points $(3,2)$, $(6,3)$, (x,y) and $(6,5)$ when joined in order and form a parallelogram, then calculate (x,y)

Soln:



'O' is midpoint of BD

$\therefore O \equiv \left(\frac{6+6}{2}, \frac{5+3}{2}\right) \equiv (6,4)$

Again 'O' is midpoint of AC

$O \equiv \left(\frac{3+x}{2}, \frac{2+y}{2}\right)$

comparing $\frac{3+x}{2} = 6$, $\frac{2+y}{2} = 4$

$x = 12-3$, $y = 8-2$

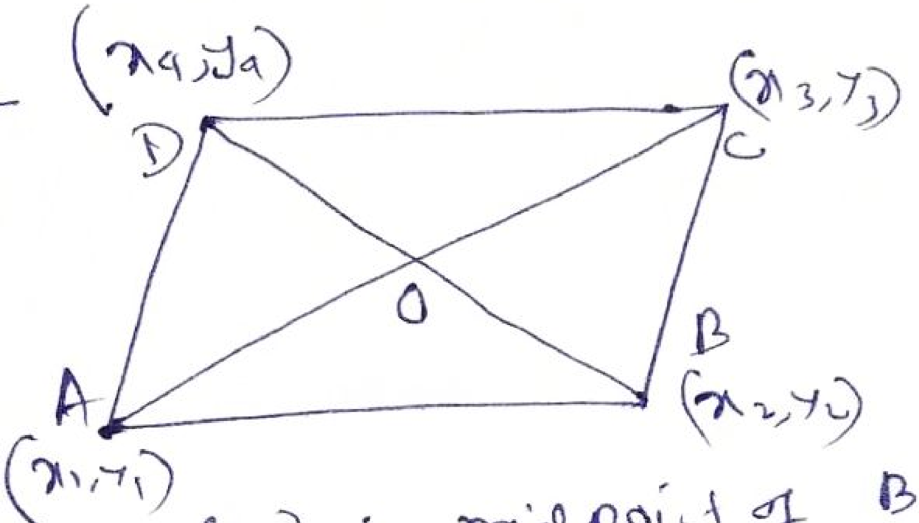
$x = 9$, $y = 6$

$\therefore (x,y) = (9,6)$ (Ans

7) If $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ & (x_4, y_4) points are joined in order to form a parallelogram, then prove that

$$x_1 + x_3 = x_2 + x_4 \quad \text{and} \quad y_1 + y_3 = y_2 + y_4$$

Soln:-



'O' is midpoint of BD

$$\therefore O \equiv \left(\frac{x_4 + x_2}{2}, \frac{y_4 + y_2}{2}\right)$$

Again 'O' is midpoint of AC

$$\therefore O \equiv \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

Comparing we get,

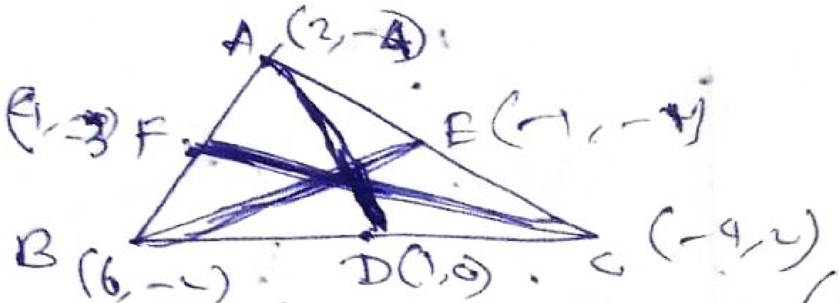
$$\frac{x_1 + x_3}{2} = \frac{x_4 + x_2}{2} \quad \& \quad \frac{y_4 + y_2}{2} = \frac{y_1 + y_3}{2}$$

$$\therefore x_1 + x_3 = x_4 + x_2 \quad \& \quad y_4 + y_2 = y_1 + y_3$$

(Ans)

9) The coordinates of vertices of triangles are $(2,-4)$, $(6,-2)$ and $(-4,2)$ respectively. Find the lengths of 3 medians of triangle.

Soln :-



D is midpoint of BC $\therefore D \equiv \left(\frac{6-4}{2}, \frac{-2+2}{2}\right)$

$\equiv (1,0)$

E is midpoint of AC $\therefore E \equiv \left(\frac{2-4}{2}, \frac{-4+2}{2}\right)$

$\equiv (-1,-1)$

F is midpoint of AB: $F \equiv \left(\frac{2+6}{2}, \frac{-4-2}{2}\right)$

$\equiv (4,-3)$

$AD = \sqrt{(2-1)^2 + (-4-0)^2}$

$= \sqrt{1^2 + (-4)^2} = \sqrt{17}$

$BE = \sqrt{\{6-(-1)\}^2 + (-2+1)^2} = \sqrt{50} = 5\sqrt{2}$

$CF = \sqrt{(-4-4)^2 + \{2-(-3)\}^2} = \sqrt{(-8)^2 + 5^2}$

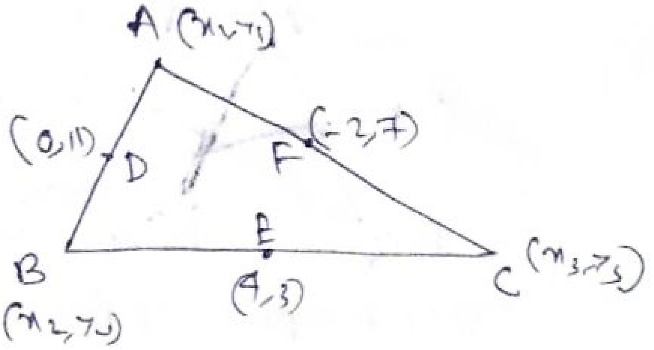
$= \sqrt{89}$

$\therefore$ length of medians

$= \sqrt{89}, 5\sqrt{2}$ & $\sqrt{17}$ units

(Ans)

10) The coordinates of midpoints of sides of a triangle are (4,3), (−2,7) and (0,11). Calculate the coordinate of vertices.



$$D \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \equiv (0,11)$$

comparing

$$x_1+x_2 = 0, \quad y_1+y_2 = 22$$

$$E \equiv \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) \equiv (4,3)$$

$$x_2+x_3 = 8, \quad y_2+y_3 = 6$$

$$F \equiv \left(\frac{x_3+x_1}{2}, \frac{y_3+y_1}{2}\right) \equiv (-2,7)$$

$$x_3+x_1 = -4, \quad y_1+y_3 = 14$$

$$x_1+x_2+x_2+x_3+x_3+x_1 = 0+8-4$$

$$2(x_1+x_2+x_3) = 4$$

$$x_1+x_2+x_3 = 2$$

$$y_1+y_2+y_2+y_3+y_1+y_3 = 22+6+14$$

$$\therefore \quad x_1+x_2+x_3 = 2$$
$$x_1+x_2 = 0$$
$$x_3 = 2$$

$$2(y_1+y_2+y_3) = 42$$
$$y_1+y_2+y_3 = 21$$
$$y_1+y_2 = 22$$
$$y_3 = -1$$

$$\therefore (x_3, y_3) \equiv (2,-1)$$

$$x_1+x_2+x_3 = 2$$
$$x_2+x_3 = 8$$
$$x_1 = -6$$

$$y_1+y_2+y_3 = 21$$
$$y_2+y_3 = 6$$
$$y_1 = 15$$

$$\therefore (x_1, y_1) \equiv (-6,15)$$

$$x_1+x_2+x_3 = 2$$
$$x_1 + x_3 = -4$$
$$x_2 = 6$$

$$y_1+y_2+y_3 = 21$$
$$y_1+y_3 = 14$$
$$y_2 = 7$$

$\therefore$ vertices are (−6,15), (6,7) and (2,−1) Ans $\quad \therefore (x_2, y_2) \equiv (6,7)$

11) (M.C.Q)

(i) The midpoint of line segment joining two points $(l, 2m)$ and $(-l+2m, 2l-2m)$ is

(a) (l, m) (b) $(l, -m)$ (c) $(m, -l)$ (d) (m, l)

Solution: Midpoint of the line joining $(l, 2m)$ and $(-l+2m, 2l-2m)$ is:

$$= \left(\frac{l - l + 2m}{2}, \frac{2m + 2l - 2m}{2}\right)$$

$$= (m, l)$$

$\therefore$ option (d)

(ii) The abscissa at the point P which divides the line segment joining two points $A(1, 5)$ and $B(-4, 7)$ internally in the ratio $2:3$ is:

(a) -1 (b) 11 (c) 1 (d) -11

Soln: The point P divides AB into $2:3$ ratio, ~~lies on~~ ~~x axis~~.

$$\therefore P \equiv \left(\frac{2\times(-4) + 3\times1}{2+3}, \frac{2\times7 + 3\times5}{2+3}\right)$$

$$\equiv \left(\frac{-8+3}{5}, \frac{29}{5}\right) \equiv \left(-1, \frac{29}{5}\right)$$

$\therefore$ abscissa at point $P = -1$

$\therefore$ option (a) correct

(iii) The coordinates of endpoints of a diameter of a circle are $(7, 9)$ and $(-1, -3)$. The coordinates of the centre of circle is:- (a) $(3, 3)$ (b) $(4, 6)$ (c) $(3, -3)$ (d) $(4, -6)$

Soln: Coordinates of centre of the circle

$$= \left(\frac{7-1}{2}, \frac{9-3}{2}\right) = \left(\frac{6}{2}, \frac{6}{2}\right) = (3, 3)$$

$\boxed{\therefore \text{option (a) correct}}$

(IV) A point which divides the line segment joining two points (2, −5) and (−3, −2) externally in the ratio 4:3. The ordinate of point is:

(a) −18 (b) −7 (c) 18 (d) 7

Soln: The point divides the line joining (2, −5) & (−3, −2) externally $\equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

$$\equiv \left(\frac{4\times(-3) - 3\times 2}{4-3}, \frac{4\times(-2) - 3\times(-5)}{4-3}\right)$$

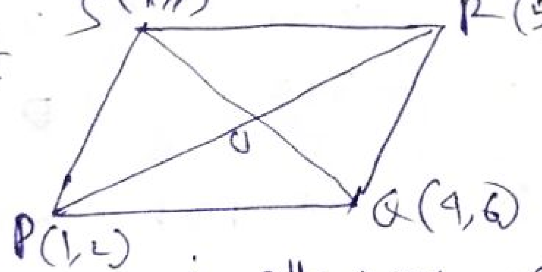
$$\equiv \left(\frac{-12-6}{1}, \frac{-8+15}{1}\right)$$

$$\equiv (-18, 7)$$

∴ ordinate of the point = 7 option (d) correct

(V) If the points P(1, 2), Q(4, 6), R(5, 7) and S(x, y) are the vertices of a parallelogram PQRS, then (a) x = 2, y = 4 (b) x = 3, y = 4 (c) x = 2, y = 3 (d) x = 2, y = 5

Soln:



In a llgm diagonals bisect each other at O.

$\therefore O \equiv \left(\frac{x+4}{2}, \frac{y+6}{2}\right)$

in other way $O \equiv \left(\frac{1+5}{2}, \frac{2+7}{2}\right)$

$\equiv \left(3, \frac{9}{2}\right)$

∴ comparing result $\frac{x+4}{2} = 3 \Rightarrow x + 4 = 6$

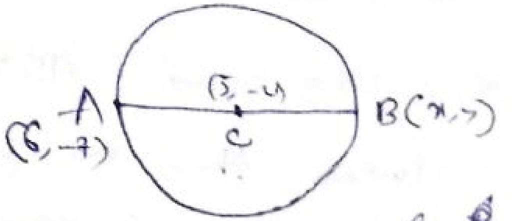
$x = 2$

$\frac{y+6}{\cancel{2}} = \frac{9}{\cancel{2}} \Rightarrow y = 3$

∴ option (c) correct

12) Short answer type questions

(i) C is the centre of a circle and AB is the diameter, the coordinates of A and C are $(6,-7)$ and $(5,-2)$. Calculate the coordinates of B.



As C is the centre,
$\therefore$ C is the midpoint of AB

$C \equiv \left(\frac{6+x}{2}, \frac{-7+y}{2}\right) \equiv (5,-2)$

Comparing we get,

$\frac{6+x}{2} = 5, \quad \frac{-7+y}{2} = -2$

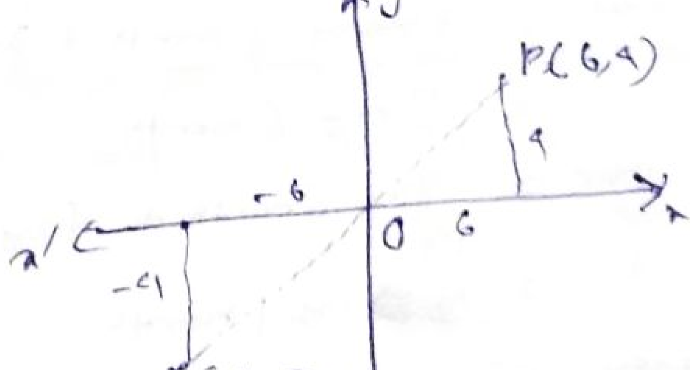
$\Rightarrow x+6 = 10, \quad -7+y = -4$

$x = 4, \quad y = 3$

$\therefore$ Coordinates of $B \equiv (4,3)$ (Ans)

(ii) The points P and Q lie on 1st and 3rd quadrant respectively. The distance of the two points from x axis and y axis are 6 units and 4 units respectively. Write the coordinates of midpoint of line segment PQ.

Soln



midpoint of PQ

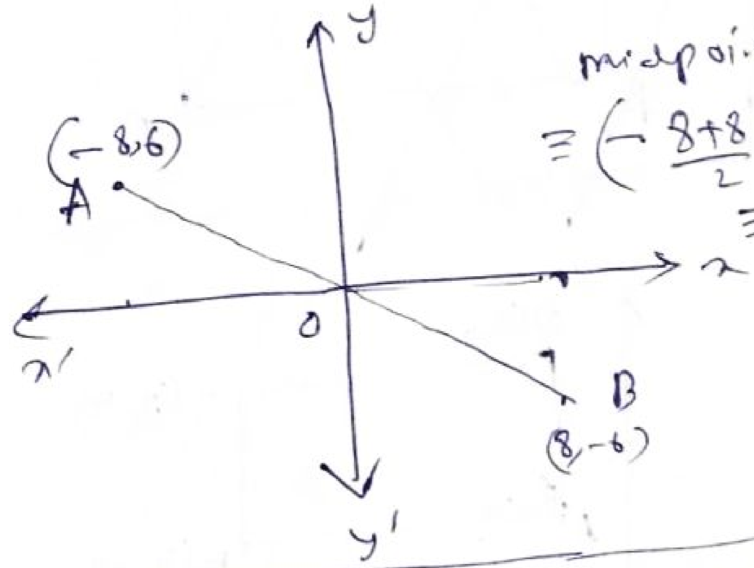
$= \left(\frac{6-6}{2}, \frac{4-4}{2}\right)$

$= (0,0)$ (Ans)

~~45) Out of 11 cows are worth Rs 25 was~~
~~removed~~

(III) The points A and B lie on 2nd and 4th quadrant respectively and distance of each point from x axis and y axis are 8 units and 6 units respectively. The coordinates of midpoint of the line segment AB

Soln



midpoint of AB

$\equiv \left(\frac{-8+8}{2}, \frac{6-6}{2}\right)$

$\equiv (0,0)$

(Ans)

(IV) The point P lies on the line segment AB and AP = PB, the coordinates of A and B are (3, -9) and (-5, 2) respectively. ~~find~~ write the coordinate of point P

Soln: As AP = PB, ∴ P is midpoint of AB.

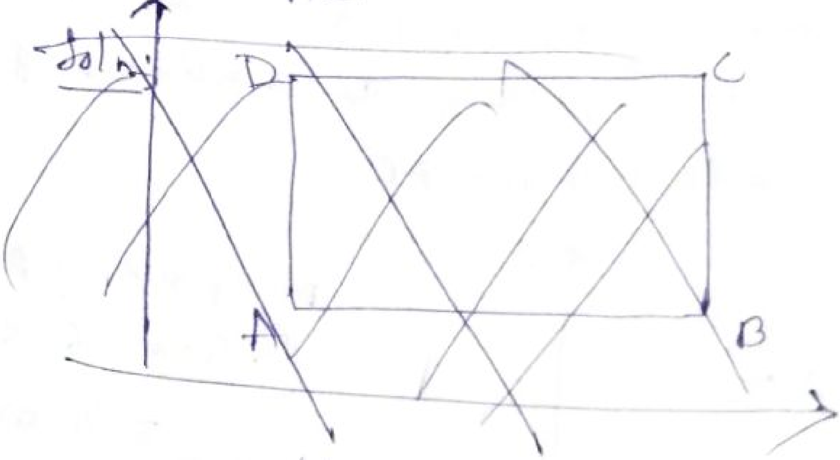
$\therefore P \equiv \left(\frac{3-5}{2}, \frac{-9+2}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right)$

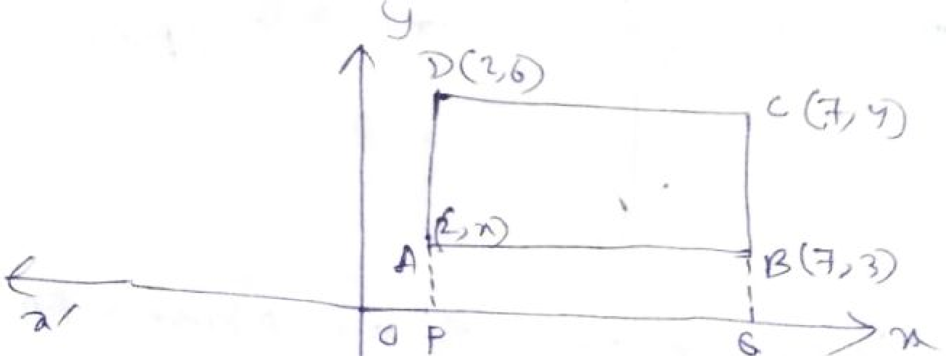
$= (-1, -1)$ (Ans)

(V)

(v) The sides of rectangle ABCD are parallel to the coordinate axes. Coordinate of B and D are (7, 3) and (2, 6). Write the coordinates of A and C and midpoint of diagonal AC.

Soln:





x component of D = OP = 2

x component of C = OQ = 7

∴ CD = AB = OQ − OP = 7 − 2 = 5

y'

$AB = \sqrt{(2-7)^2+(x-3)^2} = 5$

$\Rightarrow \sqrt{25+(x-3)^2} = 5$

$(x-3)^2 + 25 = 25$

$(x-3)^2 = 0 \Rightarrow x = 3$

$\therefore A \equiv (2, 3)$

$\therefore AD = 6 - x = 6 - 3 = 3$

$\therefore BC = 3$

but

$BC = \sqrt{(7-7)^2+(y-3)^2}$

$\therefore \sqrt{(7-7)^2+(y-3)^2} = 3$

$(y-3)^2 = 9 \Rightarrow y - 3 = 3$

$y = 6$

$\therefore C \equiv (7, 6)$

$\therefore$ midpoint of $AC \equiv \left(\frac{2+7}{2}, \frac{3+6}{2}\right) \equiv \left(\frac{9}{2}, \frac{9}{2}\right)$

$\therefore A \equiv (2, 3)$, $C \equiv (7, 6)$ & midpt $\equiv (9/2, 9/2)$ (Ans)