Cl-XII (Maths).

Significance of derivatives as rate of change.

S.M-1

Find the increment and differential of the function $y = x^2 - 2x + 3$ when x changes from 2 to 2.02.

Soln $y = x^2 - 2x + 3 \therefore \frac{dy}{dx} = 2x - 2$.

$\therefore \Delta x = 2.02 - 2 = .02$ and $dy = \left[\frac{dy}{dx}\right]_{x=2} dx = (2 \times 2 - 2) \times .02 = .04$

$\therefore$ increment $\Delta y = f(x + \Delta x) - f(x)$

$= (2.02)^2 - 2 \times 2.02 + 3 - 2^2 + 2.2 - 3$

$= 4.0804 - 4.04 + 3 - 4 + 4 - 3 = .0404$.

$\therefore$ increament $= .0404$ and differential $= .04$.

S.M-2

A spherical toy ballon is inflated so that its volume v (in cm^3) and surface area S (in cm^2) are functions of time t (in sec) where $v = \frac{\pi}{6}t^3$ and $S = \pi t^2$. Find the rates of change v and S at $t = 4$.

Soln. $v = \frac{\pi}{6}t^3 \therefore \left.\frac{dv}{dt}\right]_{t=4} = \frac{\pi}{6}3t^2 = \frac{\pi}{6}3 \cdot 4^2 = 8\pi\ cm^3/sec$.

$\therefore S = \pi t^2 \quad \left.\frac{dS}{dt}\right]_{t=4} = 2\pi t\Big|_{t=4} = 8\pi\ cm^2/sec$.

S.M-3

If the rate of change of y w.r.to x is 4 and y is changing at the rate of 12 units/sec. Find the rate of change of x per seecond.

Soln. $\frac{dy}{dx} = 4$, $\frac{dy}{dt} = 12$. $\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow 4 = \frac{12}{\frac{dx}{dt}}$

$\therefore \frac{dx}{dt} = 3$ units/sec.

S.M-4

Let v and S be the volume and surface respectively of a sphere of radius r. prove that $2\frac{dv}{dt} = r\frac{dS}{dt}$.

Soln.. $v = \frac{4}{3}\pi r^3 \quad S = 4\pi r^2 \quad \therefore \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ and $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{1}{8\pi r}\frac{dS}{dt} \quad \therefore \frac{dv}{dt} = 4\pi r^2 \cdot \frac{1}{8\pi r}\frac{dS}{dt} = \frac{r}{2}\frac{dS}{dt}$

$\therefore 2\frac{dv}{dt} = r\frac{dS}{dt}$.

S.A-5 If the error in measuring a side of a cube is 5%, find the percentage error in the computation of its volume.

Soln. Now $v = x^3$, where v = volume and x = side of the cube.

$\therefore \frac{dv}{dx} = 3x^2$. $\Delta v = \frac{dv}{dx}\Delta x$

$\therefore \Delta v = 3x^2 \Delta x$

$\therefore \frac{\Delta v}{v} = \frac{3x^2 \Delta x}{x^3} = \frac{3\Delta x}{x}$

$\therefore \frac{\Delta v}{v} \times 100 = 3\frac{\Delta x}{x} \times 100$

Now given that $\frac{\Delta x}{x} \times 100 = 5\%$.

$\therefore \frac{\Delta v}{v} \times 100 = 3 \times 5 = 15$

$\therefore$ percentage error = 15%.

S.A-6 If the area of a circle changes uniformly w.r.to time Show that the rate of change of its circumference varies inversely as its radius.

Soln. Let the radius be r unit, circumference = S unit and area = A sq unit.

Now suppose $\frac{dA}{dt} = K$ (const)

$\therefore A = \pi r^2 \quad \therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\Rightarrow K = 2\pi r \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{K}{2\pi r}$.

Again $S = 2\pi r$

$\Rightarrow \frac{ds}{dt} = 2\pi \frac{dr}{dt} = 2\pi \cdot \frac{K}{2\pi r} \Rightarrow \frac{ds}{dt} = \frac{K}{r}$

$\Rightarrow \frac{ds}{dt} \propto \frac{1}{r}$.

(7) If γ be the increase in volume of the cube of unit volume and β be the increase in area of each surface of the cube, show that $2\gamma = 3\beta$.

Soln.

Suppose each side of cube $= x$ unit, area of each surface $= S$ squnit and volume $= V$ cubeunit

$$\therefore \frac{dV}{dt} = \gamma \text{ and } \frac{dS}{dt} = \beta$$

now $V = x^3$ and $S = x^2$

$$\therefore V = S^{3/2} \text{ or } V^2 = S^3.$$

$$\Rightarrow 2V\frac{dV}{dt} = 3S^2\frac{dS}{dt}$$

$$\Rightarrow 2x^3\gamma = 3x^4\beta$$

Now putting $x = 1$, we get

$2\gamma = 3\beta$. proved.

S.HE 8.

O is a fixed pt on the st. line traced out by a moving particle. If the distance of the particle from O at time t be $(a\cos nt + b\sin nt)$ [a, b and n are constant] Show that the acceleration of the particle varies as its distance from O.

Soln. Let the distance of the particle from the pt O be x unit.

$$\therefore x = a\cos nt + b\sin nt$$

$$\frac{dx}{dt} = -an\sin nt + bn\cos nt$$

$$\therefore \frac{d^2x}{dt^2} = -an^2\cos nt - bn^2\sin nt$$

$$\Rightarrow \frac{d^2x}{dt^2} = -(an^2\cos nt + bn^2\sin nt)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -x.n^2 \quad \therefore \frac{d^2x}{dt^2} \propto x.$$

Here velocity of the particle $= \frac{dx}{dt}$

and Acceleration $= \frac{d^2x}{dt^2}$.